Effects of Join Order

Plan 1:

\[ \text{sel}_{C \times B} := \frac{|C \times B|}{|C| \times |B|} = \frac{100,000}{1,000 \times 1,000} = 0.1 \]

Plan 2:

\[ \text{sel}_{A \times B} := \frac{|A \times C|}{|A| \times |C|} = \frac{100}{1,000 \times 1,000} = 0.0001 \]

Plan 1: Top-level join has to process 1,000 + 100,000 tuples.

Plan 2: Top-level join has to process 100 + 1,000 tuples.
Cost-Based Optimization: Overall Idea
Cost-Based Optimization: Overall Idea

enumerate set of all plan alternatives
Cost-Based Optimization: Overall Idea

enumerate set of all plan alternatives

**estimate** costs of each plan
Cost-Based Optimization: Overall Idea

enumerate set of all plan alternatives

*estimate* costs of each plan

pick plan with lowest *estimated* costs
Cost-Based Optimization: Overall Idea

enumerate set of all plan alternatives

estimate costs of each plan

pick plan with lowest estimated costs

done!
Search Space for Left-Deep Trees

\[
\text{Total: } \frac{120}{24} \text{ valid order}
\]
Not a Left-Deep Plan

\[ \rightarrow \text{busy plan} \not\Rightarrow \text{left-deep plan} \]
Search Space for Bushy Trees
Search Space for Bushy Trees

2 options

1 option

2 options

2 options → 5 options
Search Space for Bushy Trees

5 options

Solution

2 options

2 options

5 options
Catalan Numbers

\[ C_n = \frac{1}{n + 1} \binom{2n}{n} = \frac{(2n)!}{(n + 1)n!n!} = \frac{(2n)!}{(n + 1)!n!} \]

\[ \binom{2n}{n} = \frac{(2n)!}{(n!)^2} \]

\[ C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} \quad \text{for} \quad n \geq 0 \]

\[ C_1 = \sum_{i=0}^{n=0} C_i C_{n-i} = C_0 \cdot C_0 = 1 \cdot 1 = 1 \]

\[ C_2 = \sum_{i=0}^{n=1} C_i C_{n-i} = C_0 \cdot C_1 + C_1 \cdot C_0 = 1 + 1 = 2 \]

\[ C_3 = \sum_{i=0}^{n=2} C_i C_{n-i} = C_0 \cdot C_2 + C_1 \cdot C_1 + C_2 \cdot C_0 = 2 + 1 + 2 = 5 \]

\[ C_4 = \sum_{i=0}^{n=3} C_i C_{n-i} = C_0 \cdot C_3 + C_1 \cdot C_2 + C_2 \cdot C_1 + C_3 \cdot C_0 = 5 + 2 + 2 + 5 = 14 \]
Search Space for Bushy Trees with 5 Input Relations

\[ C_{n-k} = C_4 = 14 \]

\[ n! \binom{2n-2}{n-2} \]

\[ = (n-1)! \left( \frac{(2n-2)!}{(n-1)!(2n-2-(n-1))!} \right) \]

\[ = \frac{(2n-2)!}{(n-1)!} \]
Search Space for Bushy Trees with 3 Input Relations

\[ n = 3 \]

\[ \frac{(2n-2)!}{(n-1)!} = \frac{4!}{2!} = 4 \cdot 3 = 12 \]

Plan 1: A C B

Plan 2: A B C

Plan 3: C B A
And the Difference is?

Plan 1:

\[ \begin{align*}
A & \rightarrow X \\
& \rightarrow 100,000 \\
& \rightarrow 1,000 \\
& \rightarrow 1,000 \\
\end{align*} \]

\( \{ 1 \} \ C \bowtie \{ 5 \} \)

\( \{ 2 \} \ (C \bowtie \{ 1 \}) \bowtie A \)

Plan 3:

\[ \begin{align*}
A & \rightarrow X \\
& \rightarrow 100,000 \\
& \rightarrow 1,000 \\
& \rightarrow 1,000 \\
\end{align*} \]

\( \{ 1 \} \ C \bowtie \{ 5 \} \)

\( \{ 2 \} \ (C \bowtie \{ 1 \}) \bowtie A \)